

APL64: MathNet Interface

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Overview

The APL64 □ MathNet system function is an interface to the [MathNet.Numerics.LinearAlgebra](#) toolkit.

Syntax

Result ← □ MathNet Action [ActionArgs]

Result: result of the specified action. Refer to □ MathNet '?' for a summary of result types

Action: Case insensitive text indicating the user-selected action

ActionArgs optional, depending on the selected action.

For ☐ MathNet mathematical actions with a rank-2 matrix argument:

- A scalar input will be processed as a 1 x 1 matrix
- A vector input will be processed as a (vector length x 1) matrix

For ☐ MathNet mathematical actions with a rank-1 vector argument:

- A scalar input will be processed as a 1-element vector
- A matrix input will be processed as its row-order ravel

Numerical Algorithms

MathNet uses numerical algorithms to obtain results. Since these algorithms are implemented on digital computers, in some cases the results obtained may be approximations of the theoretical mathematical results.

Numerically Derived Matrix Inverse

The [matrix inverse](#) of an invertible, square matrix with integer elements can have floating point elements. The matrix inverse obtained via MathNet may provide approximate element values. The theoretical mathematical definition of the matrix inverse of the matrix M is the matrix, INV which satisfies:

$$(M+. \times INV) \equiv \text{Identity matrix}$$

$$(INV+. \times M) \equiv \text{Identity matrix}$$

$$\text{identity matrix is } (IN) \circ . = \iota N \leftarrow 1 \uparrow \rho M$$

This example illustrates that the MathNet inverse for the matrix M1 is an approximation which depending on the application system may be sufficient.

```

0      M1←2 2p1.25 12 ^3 4
1      M1
2      1.25 12
3      ^3      4
4      □←INV←□MathNet 'Inverse' M1
5      0.09756097561 ^0.2926829268
6      0.07317073171 0.03048780488
7      M1+.×INV
8      1 5.551115123E^-17
9      0 1.000000000E0
10     (i2)°. = i2
11     1 0
12     0 1
13

```

Ready | Hist: Ln: 13 Col: 6 | Ins | Classic | Num | EN_US

```

M1←2 2p1.25 12 ^3 4
M1
□←INV←□MathNet 'Inverse' M1
M1+.×INV
(i2)°. = i2

```

□ MathNet Actions in APL64:

ConditionNumber

```

0      M←2 3p1 6
1      M
2      1 2 3
3      4 5 6
4      □MathNet 'ConditionNumber' M
5      12.3022455
6      A←2 2p4.1 2.8 9.7 6.6
7      A
8      4.1 2.8
9      9.7 6.6
10     □MathNet 'ConditionNumber' A
11     1622.999384
12

```

Ready | Hist: Ln: 12 Col: 6 | Ins | Classic | Num | EN_US

```

M←2 3p16
M
⎕MathNet 'ConditionNumber' M
A←2 2p4.1 2.8 9.7 6.6
A
⎕MathNet 'ConditionNumber' A

```

Determinant of a square matrix

```

APL64: CLEAR WS
File Edit Session Objects Tools Options Help
0 M←2 2p1.25 112 ^3 4
1 M
2 1.25 112
3 ^3 4
4 ⎕MathNet 'Determinant' M
5 341
6
Ready | Hist: Ln: 6 Col: 6 | Ins | Classic | Num | EN_US

```

```

M←2 2p1.25 112 ^3 4
M
⎕MathNet 'Determinant' M

```

Determinant of non-square matrix does not exist

```

APL64: CLEAR WS
File Edit Session Objects Tools Options Help
0 M←2 3p16
1 M
2 1 2 3
3 4 5 6
4 ⎕MathNet 'Determinant' M
5 LENGTH ERROR: ⎕MATHNET: right argument element : Not square
6 [imm] ⎕MathNet 'Determinant' M
7 ^
8
Ready | Hist: Ln: 8 Col: 6 | Ins | Classic | Num | EN_US

```

Diagonal

The screenshot shows the APL64: CLEAR WS window. The menu bar includes File, Edit, Session, Objects, Tools, Options, and Help. The toolbar contains various icons for file operations, editing, and execution. The main text area displays the following code:

```
0 M←2 3p16
1 M
2 1 2 3
3 4 5 6
4 □MathNet 'Diagonal' M
5 1 5
6 M1←3 3p19
7 M1
8 1 2 3
9 4 5 6
10 7 8 9
11 □MathNet 'Diagonal' M1
12 1 5 9
13 |
```

The status bar at the bottom shows 'Ready', 'Hist: Ln: 13 Col: 6', 'Ins', 'Classic', 'Num', and 'EN_US'.

Frobenius Norm

The screenshot shows the APL64: CLEAR WS window. The menu bar includes File, Edit, Session, Objects, Tools, Options, and Help. The toolbar contains various icons for file operations, editing, and execution. The main text area displays the following code:

```
0 M←2 3p16
1 M
2 1 2 3
3 4 5 6
4 □MathNet 'FrobeniusNorm' M
5 9.539392014
6 |
```

The status bar at the bottom shows 'Ready', 'Hist: Ln: 6 Col: 6', 'Ins', 'Classic', 'Num', and 'EN_US'.

Help

Same as `□MathNet '?'`

```

APL64: CLEAR WS
File Edit Session Objects Tools Options Help
0  MathNet '?'
1  MATHNET Documentation
2  Double + MATHNET "ConditionNumber" Rmatrix
3  Double + MATHNET "Determinant" Smatrix
4  Double[] + MATHNET "Diagonal" Rmatrix
5  Double + MATHNET "FrobeniusNorm" Rmatrix
6  Char[:] + MATHNET "Help"
7  Double[,] + MATHNET "Inverse" Smatrix
8  Bool + MATHNET "IsSymmetric" Rmatrix
9  Double + MATHNET "L1Norm" Rmatrix
10 Double + MATHNET "L2Norm" Rmatrix
11 Double[,] + MATHNET "LowerTriangle" Rmatrix
12 Double[,] + MATHNET "PseudoInverse" Rmatrix
13 Uvector + MATHNET "Solve" (Smatrix Kvector)
14 Uvector + MATHNET "SolvePI" (Rmatrix Kvector)
15 Uvector + MATHNET "SolveQR" (Smatrix Kvector)
16 Double[,] + MATHNET "StrictlyLowerTriangle" Rmatrix
17 Double[,] + MATHNET "StrictlyUpperTriangle" Rmatrix
18 Double[,] + MATHNET "UpperTriangle" Rmatrix
19 Char[:] + MATHNET "?"
20
21 Smatrix : Rank-2 square matrix with numeric elements: #rows = #columns
22 Rmatrix : Rank-2 rectangular or square matrix with numeric elements
23 Kvector : Known vector (double[]) sought in linear equations matrix+.xUvector = Kvector
24 Uvector : Unknown vector (double[]) sought in linear equations matrix+.xUvector = Kvector
25 PseudoInverse : More-Penrose pseudo inverse
26 Solve : Applies to square matrix
27 SolveQR : Uses QR factorization
28 SolvePI : Uses Pseudo Inverse
29 Inverse : LU decomposition
30 PseudoInverse : Moore-Penrose
31
32

```

Inverse of a square matrix

In some cases a numerical approximation will be returned.

```

APL64: CLEAR WS
File Edit Session Objects Tools Options Help
0  M1+2 2p1.25 12 ^-3 4
1  M1
2  1.25 12
3  ^-3 4
4  [+INV+MathNet 'Inverse' M1
5  0.09756097561 ^-0.2926829268
6  0.07317073171 0.03048780488
7  M1+.xINV
8  1 5.551115123E^-17
9  0 1.000000000E0
10 (i2)0.=i2
11 1 0
12 0 1
13

```

```

M←2 2p1.25 12 `3 4
M
⊖←INV←⊖MathNet 'Inverse' M1
M1+.×INV
(l2)∘.=l2

```

Argument must be a square matrix or convertible to a square matrix:

```

0      p⊖MathNet 'Inverse' 2
1      0.5
2      1 1
3      ⊖MathNet 'Inverse' (2 0 0 2)
4      LENGTH ERROR: MATHNET: right argument element : Not square
5      [imm] ⊖MathNet 'Inverse' (2 0 0 2)
6      ^
7

```

IsSymmetric

```

0      M←3 3p1 9
1      M
2      1 2 3
3      4 5 6
4      7 8 9
5      ⊖MathNet 'IsSymmetric' M
6      0
7      M←2 3p1 6
8      M
9      1 2 3
10     4 5 6
11     ⊖MathNet 'IsSymmetric' M
12     0
13     M←3 3p1 7 3 7 4 5 3 5 1
14     M
15     1 7 3
16     7 4 5
17     3 5 1
18     ⊖MathNet 'IsSymmetric' M
19     1

```

```

M←3 3p1 7 3 7 4 5 3 5 1
⊖MathNet 'IsSymmetric' M

```

L1Norm

The screenshot shows the APL64: CLEAR WS window. The menu bar includes File, Edit, Session, Objects, Tools, Options, and Help. The toolbar contains various icons for file operations, editing, and execution. The main workspace contains the following code and output:

```

MathNet 'L1Norm' (⊕+3 3p 1 7 3 7 4 5 3 5 1)
1 7 3
7 4 5
3 5 1
16
MathNet 'L1Norm' (⊕+2 3p16)
1 2 3
4 5 6
9

```

The status bar at the bottom indicates 'Ready' and 'Hist: Ln: 9 Col: 6 | Ins | Classic | Num | EN_US'.

M←3 3p1 7 3 7 4 5 3 5 1
☐ MathNet 'L1Norm' M

L2Norm

The screenshot shows the APL64: CLEAR WS window. The menu bar includes File, Edit, Session, Objects, Tools, Options, and Help. The toolbar contains various icons for file operations, editing, and execution. The main workspace contains the following code and output:

```

MathNet 'L2Norm' (⊕+3 3p 1 7 3 7 4 5 3 5 1)
1 7 3
7 4 5
3 5 1
12.55426792
MathNet 'L1Norm' (⊕+2 3p16)
1 2 3
4 5 6
9

```

The status bar at the bottom indicates 'Ready' and 'Hist: Ln: 9 Col: 6 | Ins | Classic | Num | EN_US'.

M←3 3p1 7 3 7 4 5 3 5 1
☐ MathNet 'L2Norm' M

Lower Triangle

```
APL64: CLEAR WS
File Edit Session Objects Tools Options Help
[Icons]
'Square Matrix:'
M←3 3p 1 7 3 7 4 5 3 5 1
M
'Lower Triangle of M:'
MathNet 'LowerTriangle' M
'Rectangular Matrix:'
M←2 3p 6
M
'Lower Triangle of M:'
MathNet 'LowerTriangle' M

Square Matrix:
1 7 3
7 4 5
3 5 1
Lower Triangle of M:
1 0 0
7 4 0
3 5 1

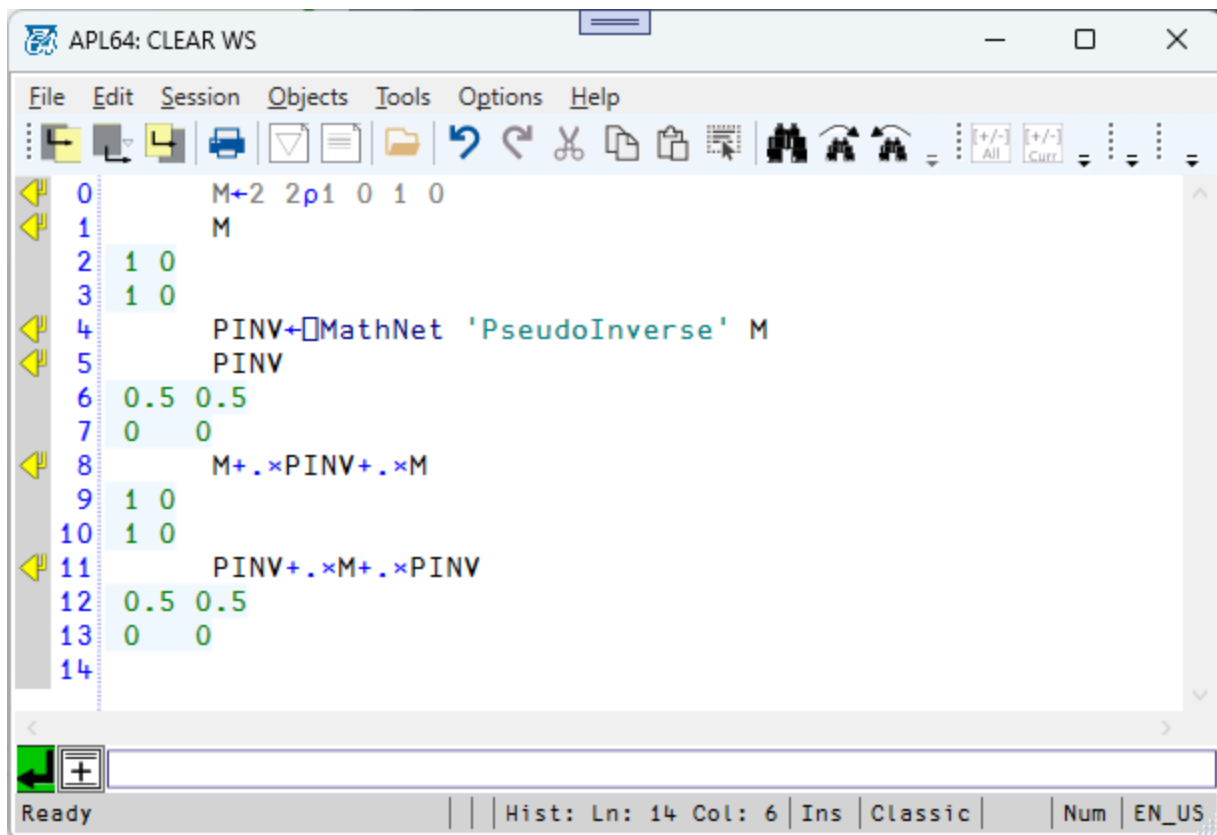
Rectangular Matrix:
1 2 3
4 5 6
Lower Triangle of M:
1 0 0
4 5 0
```

```
M←3 3p 1 7 3 7 4 5 3 5 1
M
'LowerTriangle of M: '
MathNet 'LowerTriangle' M
```

Pseudo Inverse

Any rectangular or square matrix has a [Pseudo Inverse](#).

```
APL64: CLEAR WS
File Edit Session Objects Tools Options Help
[Icons]
0 MZERO←5 5p0
1 MZERO≡MathNet 'PseudoInverse' MZERO
2 1
3 |
```



The screenshot shows the APL64: CLEAR WS window with the following content:

```
0      M←2 2p1 0 1 0
1      M
2      1 0
3      1 0
4      PINV←⎕MathNet 'PseudoInverse' M
5      PINV
6      0.5 0.5
7      0 0
8      M+.×PINV+.×M
9      1 0
10     1 0
11     PINV+.×M+.×PINV
12     0.5 0.5
13     0 0
14
```

The status bar at the bottom indicates: Ready | Hist: Ln: 14 Col: 6 | Ins | Classic | Num | EN_US

```

0      A←2 3⍴1 2 3 4 5 6
1      A
2      1 2 3
3      4 5 6
4      PINV←⎕MathNet 'PseudoInverse' A
5      PINV
6      -0.9444444444  0.4444444444
7      -0.1111111111  0.1111111111
8      0.7222222222 -0.2222222222
9      A+.×PINV
10     1 -5.551115123E-17
11     0 1.000000000E0
12     ⍠ Numerically approximate identity matrix
13     A+.×INV+.×A
14     1 2 3
15     4 5 6
16

```

Ready | Hist: Ln: 16 Col: 6 | Ins | Classic | Num | EN_US

Solve

The Solve action applies to square, non-singular (invertible) matrices.

Find x and y satisfying the system of simultaneous linear equations: $1x + 3y = 14$ and $4x + 2y = 26$

```

0      b←14 26
1      m←2 2p1 3 4 2
2      m
3      1 3
4      4 2
5      res←MathNet 'Solve' (m b)
6      res
7      5 3
8      m+.×res
9      14 26
10

```

Ready | Hist: Ln: 10 Col: 6 | Ins | Classic | Num | EN_US

Solve for X where $B = M+.xX$

```

0      M←3 3p19
1      M[3;3]←1.2345
2      B←4 5 6
3      X←MathNet 'Solve' (M B)
4      X
5      M+.×X
6      -3.333333333 3.666666667 -1.143749172E-16
7      4 5 6
8

```

Ready | Hist: Ln: 8 Col: 6 | Ins | Classic | Num | EN_US

Solve PI

The SolvePI action applies to square or rectangular matrices and uses the pseudo inverse in its algorithm.

Solve for X1, where $B = M \cdot X1$

```

APL64: CLEAR WS
File Edit Session Objects Tools Options Help
M←3 7p0
M[1;]←3 2 1 5 -1 0 0
M[2;]←2 1 1 2 0 -1 0
M[3;]←5 11 3 4 0 0 -1
M
3 2 1 5 -1 0 0
2 1 1 2 0 -1 0
5 11 3 4 0 0 -1
B←-3 -5 -2
X1←MathNet 'SolvePI' (M B)
X1
-1.326319305 0.6816967268 -0.9589178357 -0.02672010688 -0.7080828323 2.016700067 -0.1165664663
M+.×X1
-3 -5 -2

```

```

M←3 7p0
M[1;]←3 2 1 5 -1 0 0
M[2;]←2 1 1 2 0 -1 0
M[3;]←5 11 3 4 0 0 -1
M
B←-3 -5 -2
X1←MathNet 'SolvePI' (M B)
X1
M+.×X1

```

Solve for X where $B = M \cdot X$

```

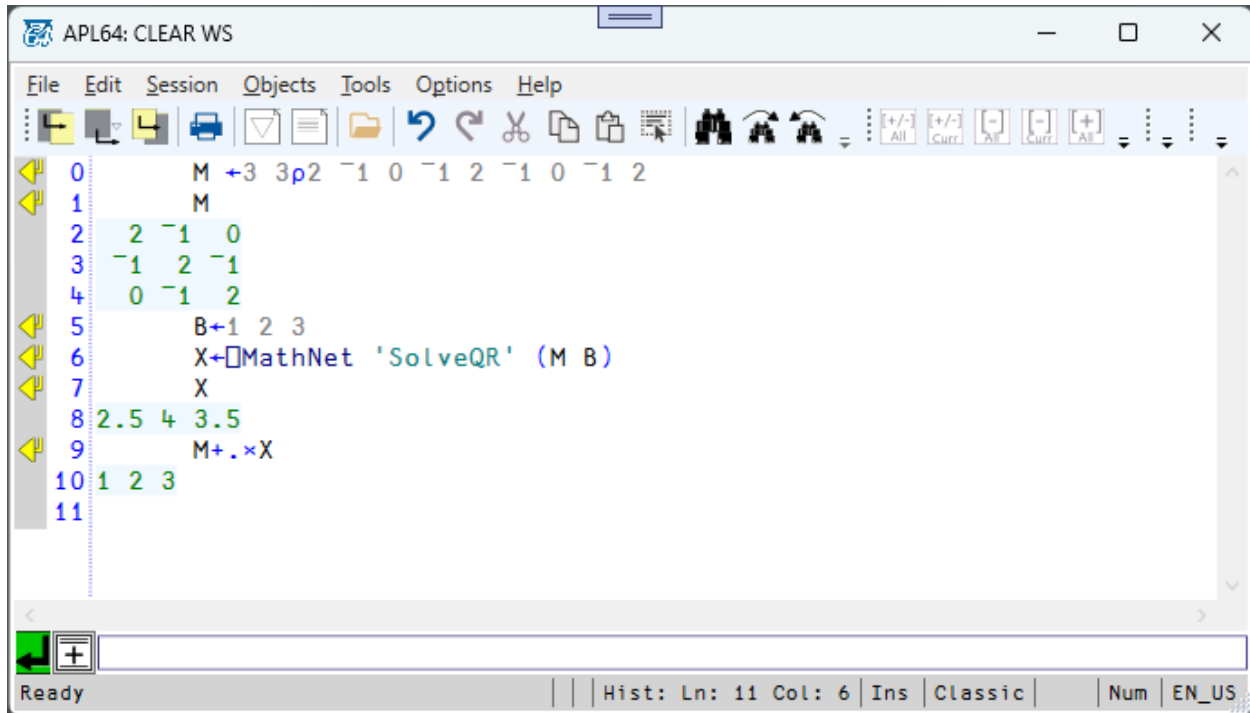
APL64: CLEAR WS
File Edit Session Objects Tools Options Help
M←3 3p19
M[3;3]←1.2345
B←4 5 6
X←MathNet 'SolvePI' (M B)
X
M+.×X
-3.333333333 3.666666667 -1.110223025E-16
4 5 6

```

Solve QR

The SolveQR action applies to square matrices and uses the QR factorization in its algorithm.

Solve for X where $B = M+. \times X$



The screenshot shows the APL64: CLEAR WS window with the following commands and results:

```
0 M ← 3 3p2 ^1 0 ^1 2 ^1 0 ^1 2
1 M
2 2 -1 0
3 -1 2 -1
4 0 -1 2
5 B ← 1 2 3
6 X ← □ MathNet 'SolveQR' (M B)
7 X
8 2.5 4 3.5
9 M+. × X
10 1 2 3
11
```

The status bar at the bottom indicates: Ready | Hist: Ln: 11 Col: 6 | Ins | Classic | Num | EN_US

```
M←3 3p2 ^1 0 ^1 2 ^1 0 ^1 2
M
B←1 2 3
X←□ MathNet 'SolveQR' (M B)
X
M+.×X
```

Solve for X where $B = M+. \times X$

```

APL64: CLEAR WS
File Edit Session Objects Tools Options Help
[Icons] [All] [Curr]
0 M←3 3p19
1 M[3;3]+1.2345
2 B←4 5 6
3 X←MathNet 'SolveQR' (M B)
4 X
5 M+.×X
6 -3.333333333 3.666666667 -1.26072084E-15
7 4 5 6
8

```

Ready | Hist: Ln: 8 Col: 6 | Ins | Classic | Num | EN_US

Strictly Lower Triangle

```

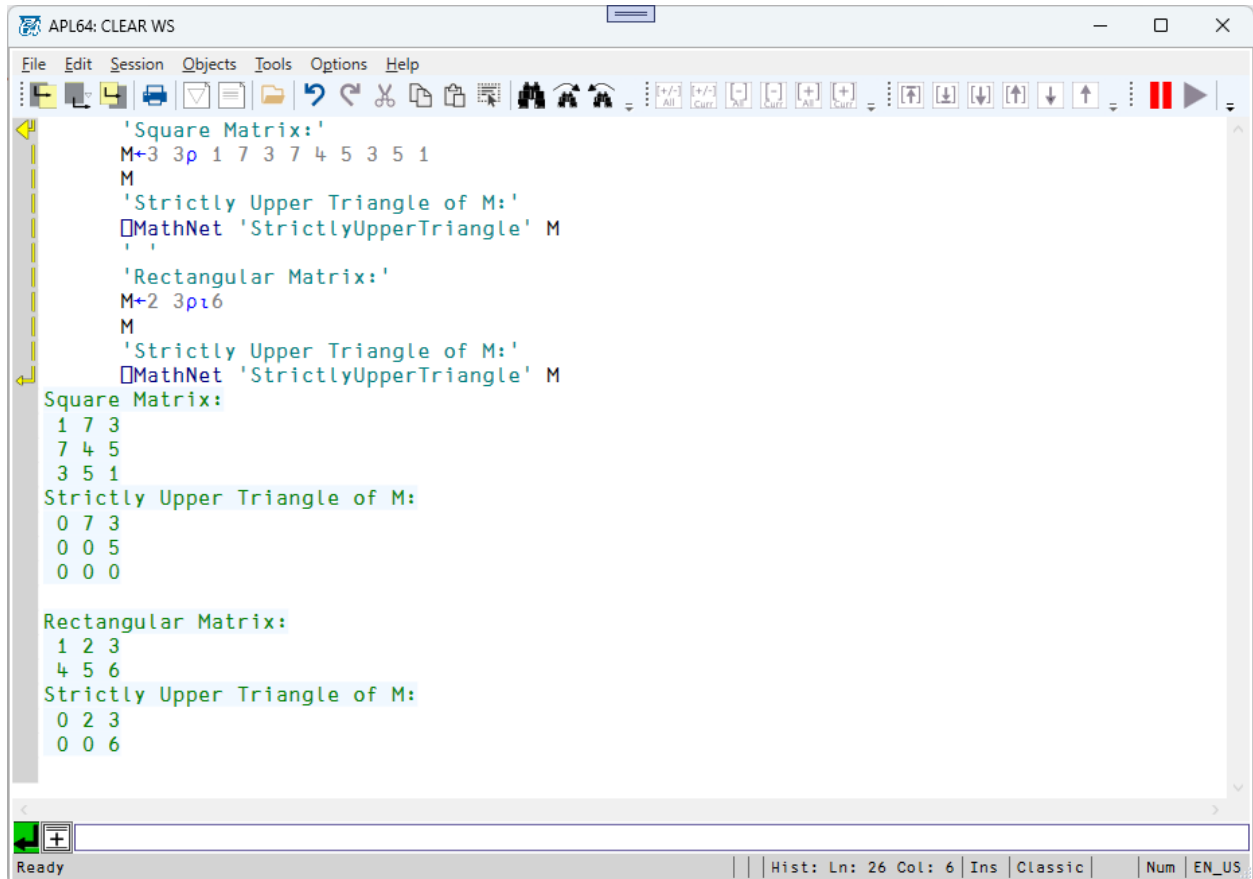
APL64: CLEAR WS
File Edit Session Objects Tools Options Help
[Icons] [All] [Curr] [X] [Curr] [All] [Curr] [Icons]
'Square Matrix:'
M←3 3p 1 7 3 7 4 5 3 5 1
M
'Strictly Lower Triangle of M:'
MathNet 'StrictlyLowerTriangle' M
'Rectangular Matrix:'
M←2 3p16
M
'Strictly Lower Triangle of M:'
MathNet 'StrictlyLowerTriangle' M
Square Matrix:
1 7 3
7 4 5
3 5 1
Strictly Lower Triangle of M:
0 0 0
7 0 0
3 5 0
Rectangular Matrix:
1 2 3
4 5 6
Strictly Lower Triangle of M:
0 0 0
4 0 0

```

Ready | Hist: Ln: 26 Col: 6 | Ins | Classic | Num | EN_US

```
M←3 3p1 7 3 7 4 5 3 5 1
```

Strictly Upper Triangle



The screenshot shows the APL64: CLEAR WS window. The code entered is as follows:

```
'Square Matrix:'  
M←3 3p 1 7 3 7 4 5 3 5 1  
M  
'Strictly Upper Triangle of M:'  
⌈MathNet 'StrictlyUpperTriangle' M  
'Rectangular Matrix:'  
M←2 3p 6  
M  
'Strictly Upper Triangle of M:'  
⌈MathNet 'StrictlyUpperTriangle' M
```

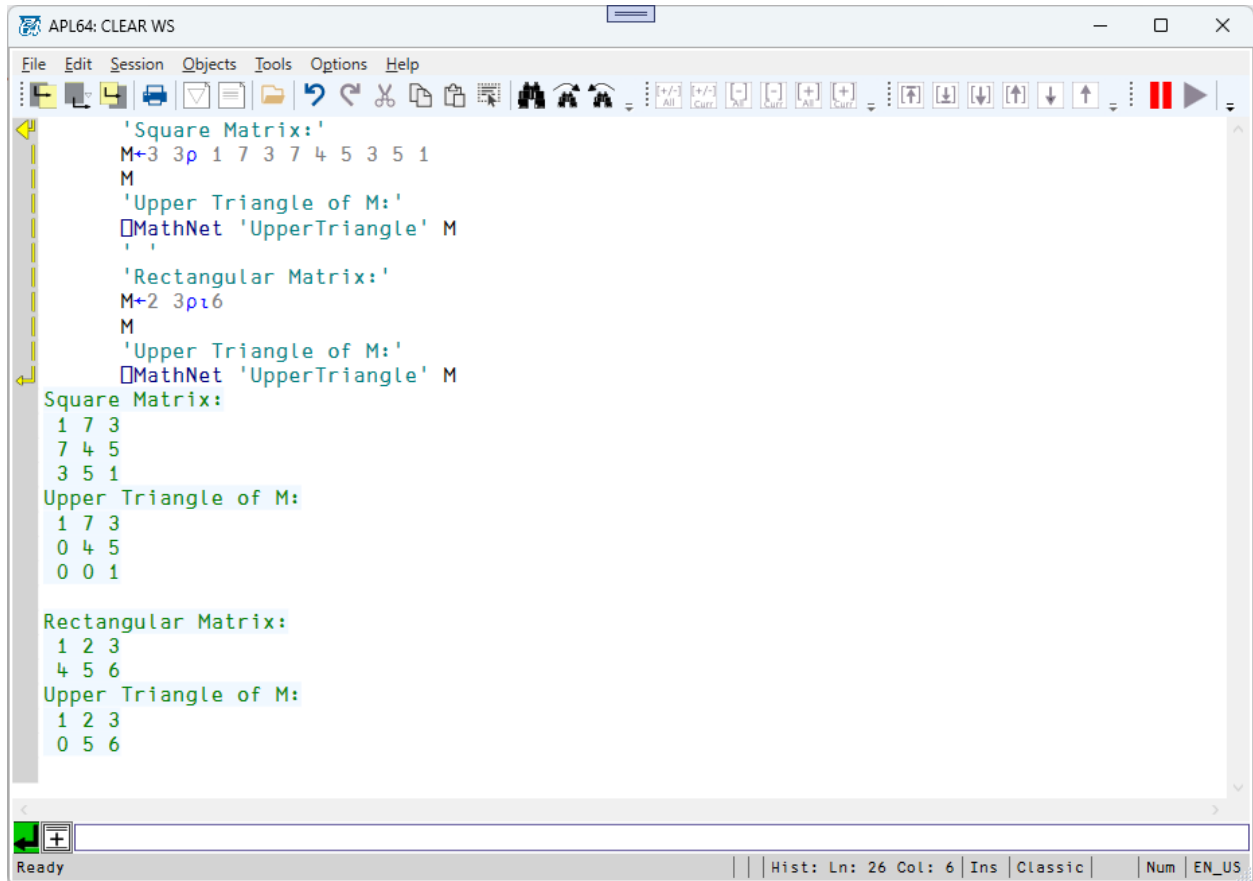
The output of the code is displayed below the code:

```
Square Matrix:  
1 7 3  
7 4 5  
3 5 1  
Strictly Upper Triangle of M:  
0 7 3  
0 0 5  
0 0 0  
  
Rectangular Matrix:  
1 2 3  
4 5 6  
Strictly Upper Triangle of M:  
0 2 3  
0 0 6
```

The status bar at the bottom indicates 'Ready' and 'Hist: Ln: 26 Col: 6 Ins Classic Num EN_US'.

M←3 3p1 7 3 7 4 5 3 5 1

Upper Triangle



The screenshot shows the APL64: CLEAR WS window with the following content:

```
'Square Matrix:'  
M←3 3p 1 7 3 7 4 5 3 5 1  
M  
'Upper Triangle of M:'  
⎕MathNet 'UpperTriangle' M  
'Rectangular Matrix:'  
M←2 3p 6  
M  
'Upper Triangle of M:'  
⎕MathNet 'UpperTriangle' M
```

The results of the operations are displayed below the code:

Square Matrix:

1	7	3
7	4	5
3	5	1

Upper Triangle of M:

1	7	3
0	4	5
0	0	1

Rectangular Matrix:

1	2	3
4	5	6

Upper Triangle of M:

1	2	3
0	5	6

The status bar at the bottom indicates: Ready | Hist: Ln: 26 Col: 6 | Ins | Classic | Num | EN_US

M←3 3p1 7 3 7 4 5 3 5 1